Capital dynamics as a trade-off between tax rate and budget deficit

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Abstract A dynamic open economic model is considered on the basis of a capital accumulation equation (with variable depreciation rate) and gross national product (GNP) as a sum of consumption, investments, net export, government expenditures. The standard assumptions are used: 1) investments decrease when real interest rate increases; 2) net export decreases with increase of GNP and real interest rate; 3) at given tax rate, budget deficit is caused by government expenditures. This model is a basic one along with the models of M.Kalecki, N.Kaldor, R.M.Goodwin and others, but didn't get due attention because tax rate and budget deficit usually were not considered as controllable instruments in an open economy. The results are: i) at marginal propensity to consume below 1, the more tax rate, the less capital growth (and sometimes more GNP); ii) the more budget surplus, the more capital growth (and sometimes less GNP). Such implications for GNP behaviour provide new patterns of economic growth with business cycles.

1. INTRODUCTION

The main purpose of the paper is analysis of capital dynamics of a national economy depending on basic fiscal economic policy instruments such as tax rate and budget deficit. The analysis is topical for transitional economies in Europe, including Germany (McDonald and Thumann [1990]) and Ukraine (McCarthy et al. [1995]), the Russian Federation and Asia (Sachs and Wing Thye Woo [1994]). It is shown that in general impacts of fiscal policy on dynamics of capital and GNP (output) are different, and in some cases these impacts are opposite.

2. THE IS-LM MODEL

The standard IS-LM model is described by the following equations in Hall and Taylor [1991]:

(1) $Y = C + I + G + X,$

(2) $C = a + b\,(Y - \tau Y),$

(3) $I = l - d\,R,$

(4) $X = g - m\,Y - n\,R,$

(5) $M / P = k\,Y - h\,R,$

(6) $E\,P / P_w = q + v\,R,$

where Y, C, I, G, X, R, M, P, E, P_w are GNP, consumption, investments, government expenditures, net export, real interest rate, money, national price level, nominal exchange rate of national currency, world price level, respectively, a, b, l, d, g, m, n, k, h, q, v are given positive coefficients, and τ is aggregate tax rate.

3. ENDOGENOUS, EXOGENOUS, AND CONTROLLABLE VARIABLES OF THE MODEL

In the model (1)-(6) variables G, M, E, P_w, τ are assumed to be exogenous, and variables Y, C, I, X, R are considered as endogenous variables. The subset of exogenous variables, the variables G, M, E, τ, is called the set of controllable variables. G and τ are fiscal controllable variables.

Let us introduce a new fiscal controllable variable

(7) $D = \tau\,Y - G$

called budget deficit which seems to be more convenient fiscal control than G. If $D > 0$, D is called surplus; otherwise it is called deficit. Henceforth D and τ will represent fiscal controllable variables in a national economy.

4. CAPITAL DETERMINATION

The capital K dynamics in time t is determined by the following differential equation according to Kaldor [1940] and Goodwin [1951]:

(8) $d\,K / d\,t = I + \delta(t)\,K,$

where $\delta(t)$ is a variable capital depreciation rate. Consequently (3) and (8) provide

(9) $$d K / d t - \delta(t) K = I - d R,$$

and $K(t) = K(I, d, \delta(t), R(t), t)$. Treating $\delta(t)$ as an exogenous variable, let us assume for simplicity of notations that $\delta(t) = \delta$ is constant. Then (9) gives

(10) $$K(t) = K(0) \exp(-\delta t) + \int_0^t \{[I - d R(u)] \exp[\delta(t-u)]\} du = K(0) \exp(-\delta t) + I (\exp \delta t - 1)/\delta - d \int_0^t R(u) \exp[\delta(t-u)] du.$$

As a result capital $K(t)$ depends on controllable variables through real interest rate $R(t)$.

5. THE REAL INTEREST RATE IN TERMS OF CONTROLLABLE VARIABLES

From (1)-(4), (7) we get

(11) $$Y = [a + g + I - D - (d + n) R] / [m + (1 - b)(1 - \tau)],$$

and from (5) we obtain

(12) $$Y = (M + h P R) / (k P).$$

Then

(13) $$k P [a + g + I - D - (d + n) R] = (M + h P R) [m + (1 - b)(1 - \tau)].$$

This together with (6) provides the equation for R in terms of controllable variables:

(14) $$k P_w (q + v R) [a + g + I - D - (d + n) R] = [M E + h P_w (q + v R) R] [m + (1 - b)(1 - \tau)].$$

After rearranging in the equation above we get the following quadratic equation for R:

(15) $$\alpha v R^2 + (q \alpha - v \beta) R + \chi - q \beta = 0,$$

where

(16) $$\alpha = P_w \{h [m + (1 - b)(1 - \tau)] + k (d + n)\},$$

(17) $$\beta = P_w k (a + g + I - D),$$

(18) $$\chi = M E [m + (1 - b)(1 - \tau)].$$

As a result, we obtain the relationship between budget deficit D and tax rate τ:

(19) $$[q \alpha(\tau) - v \beta(D)]^2 \geq 4 v \alpha(\tau) [\chi(\tau) - q \beta(D)].$$

Differentiating the inequality above, we get

(20) $$[q \alpha(\tau) - v \beta(D)] q \, \partial\alpha / \partial\tau \geq 2 v \{\alpha(\tau) \, \partial\chi / \partial\tau + \partial\alpha / \partial\tau [\chi(\tau) - q \beta(D)]\},$$

(21) $$[q \alpha(\tau) - v \beta(D)] \, \partial\beta / \partial D \leq 2 q \alpha(\tau) \, \partial\beta / \partial D.$$

From (16)-(18) we receive

(22) $$\partial\alpha / \partial\tau = P_w h (b - 1) < 0,$$

(23) $$\partial\beta / \partial D = - P_w k < 0,$$

(24) $$\partial\chi / \partial\tau = M E (b - 1) < 0$$

due to the common assumption that marginal propensity b to consume is below 1. Then (20)-(21) provide

(25) $$[q^2 \alpha(\tau) + v q \beta(D) - 2 v \chi(\tau)] \, \partial\alpha / \partial\tau \geq 2 v \alpha(\tau) \, \partial\chi / \partial\tau,$$

(26) $$q \alpha(\tau) + v \beta(D) \leq 0.$$

The latter inequality implies the new relationship between budget deficit D and tax rate τ:

(27) $$D \geq q \{h [m + (1 - b)(1 - \tau)] + k (d + n)\} / (k v) + a + g + I > 0.$$

This stands for that: i) the IS-LM model has a property of budget surplus; ii) high tax rate decreases the feasible lower limit of budget surplus.

6. CAPITAL DYNAMICS DEPENDING ON FISCAL CONTROLLABLE VARIABLES

From (10) we have

$$(28) \qquad \partial K / \partial\tau = - d \int_0^t \exp [\delta (t - u)] \, \partial R / \partial\tau \, d u,$$

$$(29) \qquad \partial K / \partial D = - d \int_0^t \exp [\delta (t-u)] \, \partial R / \partial D \, d u.$$

The partial derivatives of R are obtained from (15):

$$(30) \qquad 2 \alpha v R \, \partial R / \partial\tau + v R^2 \partial\alpha / \partial\tau + (q \alpha - v \beta) \, \partial R / \partial\tau + q R \, \partial\alpha / \partial\tau + \partial\chi / \partial\tau = 0,$$

$$(31) \qquad 2 \alpha v R \, \partial R / \partial D + (q \alpha - v \beta) \, \partial R / \partial D - v R \, \partial\beta / \partial D - q \, \partial\beta / \partial D = 0.$$

Then, taking into account (21)-(24), we get:

$$(32) \qquad \partial R / \partial\tau = - [(v R + q) \, \partial\alpha / \partial\tau + \partial\chi / \partial\tau] / (2 v \alpha + q \alpha - v \beta) > 0,$$

$$(33) \qquad \partial R / \partial D = \partial\beta / \partial D \, (v + q) / (2 v \alpha R + q \alpha - v \beta) < 0$$

at positive real interest rates.

As a result, from (28)-(29), (32)-(33) we have

$$(34) \qquad \partial K / \partial\tau < 0,$$

$$(35) \qquad \partial K / \partial D > 0.$$

As it might be expected, capital is decreasing with growth of tax rates and increasing with growth of budget surplus.

7. CAPITAL VS. OUTPUT DEPENDING ON FISCAL CONTROLLABLE VARIABLES

From (11) we obtain:

$$(36) \qquad \partial Y / \partial\tau = \{- (d + n) [m + (1 - b) (1 - \tau)] \, \partial R / \partial\tau - (b - 1) [a + g + l - D - (d + n) R]\} / [m + (1 - b) (1 - \tau)]^2$$

is positive when

$$(37) \qquad \partial R / \partial\tau < (1 - b) [a + g + l - D - (d + n) R] / \{(d + n) [m + (1 - b) (1 - \tau)]\},$$

that is when real interest rate R is slow growing at increase of tax rate τ. If real interest rate R is fast growing at increase of tax rate τ, then output Y is diminishing together with capital K.

From (11) we get:

$$(38) \qquad \partial Y / \partial D = - [1 + (d + n) \, \partial R / \partial D] / [m + (1 - b) (1 - \tau)]$$

is positive when

$$(39) \qquad \partial R / \partial D < - 1 / (d + n) < 0.$$

In this case $\partial K / \partial D$ is positive as well. In other cases $\partial K / \partial D$ is also positive while $\partial Y / \partial D$ is negative.

8. CONCLUSIONS

The results above demonstrate that capital growth is not always accompanied by output growth, especially under transitional fiscal policy.

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